Uncertainty estimation for high-resistance standards of the wye-delta type

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Abstract

Several wye-delta-type high-resistance standards between 1 MΩ to 10 GΩ having 4-terminal structures were designed. The structures can directly obtain the calculated value of resistance whenever necessary, eliminating the need to consider long-term stability. For those resistances, the expanded uncertainties were evaluated according to the uncertainty propagation law of the ISO GUM Guide. We demonstrate that the uncertainty of the high resistance to be measured can be significantly improved by reducing the measurement uncertainties of the element resistances in each range, and that expanded uncertainty for the 1 GΩ resistance having the 4-terminal structure can be obtained at the 0.1×10^{-6} level by correcting the lead resistance effect between the system GND and the element resistance with low resistance and the leakage resistance caused by the two main element resistances. The uncertainty result could be used for quantum triangle metrology (QMT) research using a 1 GΩ resistance.

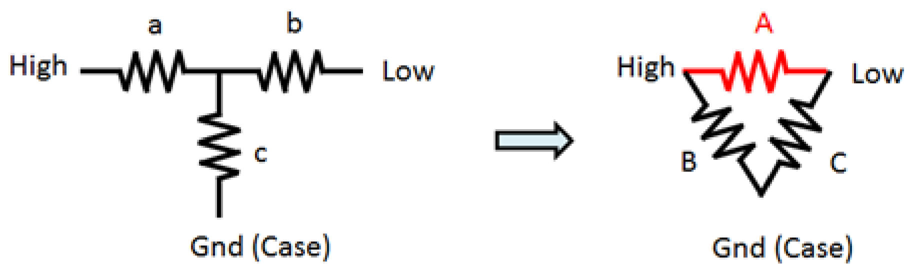
Keywords: Wye-delta-type high resistance, Potentiometric method, Modified Wheatstone bridge method, High-resistance meter, quantum triangle metrology

1 Introduction

Systems that precisely measure high resistances of 1 MΩ or more are widely used by various measuring instruments and methods according to the measurement range. These instruments and methods include the digital multimeter (DMM) that can measure up to 20 GΩ, the bridge method that can measure up to 10 PΩ [1–3], the DMM–calibrator measurement method[4], the potential difference method [4], the teraohmmeter, and others. In addition, resistors in the range of up to 10 GΩ are used to calibrate the high-resistance measurement function of meter calibrators and DMMs are used as the standard for electrical measurements. Moreover, for high-resistance standard resistances corresponding to measurement systems of 10 GΩ or more, commercial products of up to 10 PΩ are commonly used. Among these products, wye-delta-type high-resistance standard resistors that obtain a high target resistance value with transformation into a delta network form by constructing a wye network with element resistors of low-resistance value are also used. Unlike single-element high resistors, a wye-delta-type high resistor is composed of a wye network of at least three elements, where each resistor element has a relatively low resistance value, so it can be made at a low cost and presents the advantage of being capable of obtaining a high resistance with high accuracy. In this study, using these features, the uncertainty for wye-delta-type 1 MΩ ~ 10 GΩ resistances used for the main element of 100 GΩ or higher resistances and high-resistance key comparisons was evaluated according to the uncertainty propagation law of ISO GUM [5]. In particular, we wished to see if the uncertainty evaluation result for the 1 GΩ range can be used in QMT research [6, 7] to prove Ohm's law using Josephson voltage, quantum Hall resistance, and single electron current, since the 1 GΩ range can be used to lower measurement uncertainty, which is needed in QMT[16].

2 Wye-delta transformation theory

Since the equivalence theory of the wye network and the delta network was published by A.E. Kennelly in 1899 [8], wye-delta transformation has been applied to various electrical and electronic circuits as well as to various commercial products in the high-resistance range using the concept [9, 10]. According to the equivalence theory, (Fig. 1), the wye network, which is composed of the resistive elements a, b, and c, is transformed into a delta network composed of the resistance elements A, B and C, as given by equation (1)[3].



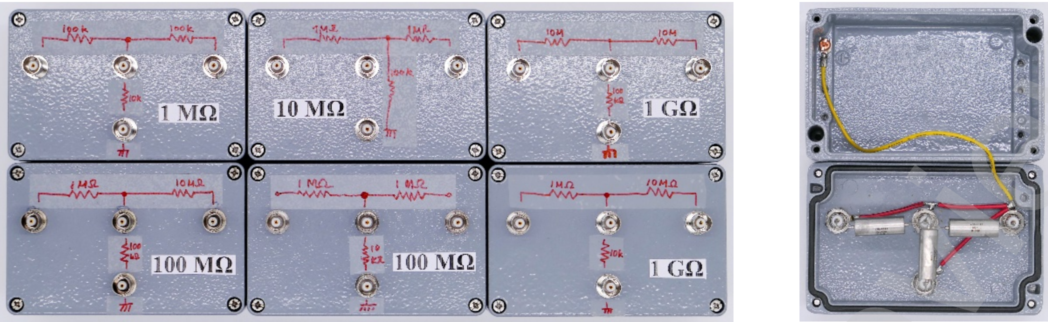
(a) Wye network (b) Delta network

Fig. 1 Wye-delta transformation

- $A = a + b + (a \times b) / c$
- $B = a + c + (a \times c) / b$
- $C = b + c + (b \times c) / a$ (1)

3 Wye-delta-type high resistance

The resistance elements a, b, and c of the wye network in Fig. 1 are composed of three-element resistances having a wye shape, and coaxial connectors are externally mounted to create a wye-delta-type high resistance (Fig. 2) [11, 12]. This configuration has the advantage of directly measuring the resistance of each element, and after obtaining the calculation value of the high resistance, the result is inserted into equation (1). The wye network can be configured using three commercially available standard resistors (Fig. 3). Commercial products composed of N-type connectors and a wye network are also used (Fig. 4). The value of these high resistances can be determined using various measurement methods such as the modified Wheatstone bridge, the potentiometric method, and the teraohmmeter, and measurement uncertainty has been steadily improving to date [13–15]. In the case of many National Metrology Institutes (NMIs), the measurement uncertainty is about 1 ppm or less, and in the case of NMIs using a quantum Hall resistance and a cryogenic current comparator bridge, the uncertainty level is close to 0.1 ppm [16]. With this situation, it is considered that investigating whether the uncertainty evaluation results are at a QMT-usable level, and whether NMIs can easily apply wye-delta-type high resistances as commercial products can have scientific and technological significance.



(a) Outer shape (b) Inner shape

Fig. 2 The wye--type high resistances (1 MΩ ~ 1 GΩ) constructed for dissemination [10]

Fig. 3 A wye-shape high resistance composed of a three-element resistance, elements a, b, and c

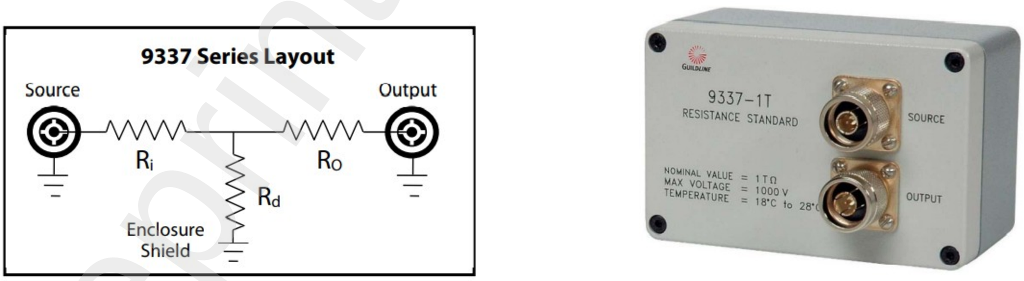


Fig. 4 Inner and outer shape of a commercial high resistor of the wye-delta type

4. Uncertainty evaluation

By the law of propagation of uncertainty in the ISO Guide for the mathematical model given by the first relation of equation (1), the relative combined standard uncertainty of an unknown resistance A is given in a linear approximation as follows equations (2–5) [5].

$$u^2(A) = \left(\frac{\partial A}{\partial a}\right)^2 \cdot u^2(a) + \left(\frac{\partial A}{\partial b}\right)^2 \cdot u^2(b) + \left(\frac{\partial A}{\partial c}\right)^2 \cdot u^2(c), \tag{2}$$

where $\frac{\partial A}{\partial a} = \frac{b}{c} + 1, \frac{\partial A}{\partial b} = \frac{a}{c} + 1$ and $\frac{\partial A}{\partial c} = -\frac{a \cdot b}{c^2}$

$$\frac{u^2(A)}{A^2} = \frac{\left(\frac{b}{c}+1\right)^2 \cdot u^2(a)}{a \cdot \left(\frac{b}{c}+1\right) + b^2} + \frac{\left(\frac{a}{c}+1\right)^2 \cdot u^2(b)}{b \cdot \left(\frac{a}{c}+1\right) + a^2} + \frac{\left(\frac{a \cdot b}{c^2}\right)^2 \cdot u^2(c)}{\left(\frac{a \cdot b}{c} + a + b\right)^2} \tag{3}$$

$$= \frac{u^2(a)}{a^2} \cdot \frac{\left(\frac{b}{c}+1\right)^2}{\left(\frac{b}{c}+1\right) + \frac{b^2}{a}} + \frac{u^2(b)}{b^2} \cdot \frac{\left(\frac{a}{c}+1\right)^2}{\left(\frac{a}{c}+1\right) + \frac{a^2}{b}} + \frac{u^2(c)}{c^2} \cdot \frac{\left(\frac{a \cdot b}{c}\right)^2}{\left(\frac{a \cdot b}{c} + a + b\right)^2} \tag{4}$$

$$\cong \frac{u^2(a)}{a^2} + \frac{u^2(b)}{b^2} + \frac{u^2(c)}{c^2} \tag{5}$$

Based on equation (2), the combined standard uncertainty and expanded uncertainty were evaluated to be in the range of 1 MΩ ~ 10 GΩ and the results are shown in Table 1.

Table 1

Uncertainty evaluation for wye-delta type standard in the 1 MΩ to 10 GΩ range

1 MΩ									
a (Ω)	b (Ω)	c’ (Ω)	$A(R_X)$ = a+b+a_x b/c’ (Ω)	B = a+c’+a_x c’/b (Ω)	$C(R_b)$ = b+c’+b_x c’/a (Ω)	*u*a (Ω)	*u*b (Ω)	*u*c’ (Ω)	u_{Rb} (ppm)
1000	1000	1	1002000	1002	1002	0.000015	0.000015	0.000000015	0.015
1000	1000	10	1011000	1011	10110	0.000015	0.000150	0.00000015	0.015
10000	10000	100	1020000	10200	10200	0.000150	0.000150	0.0000015	0.015
10000	10000	1000	1110000	11100	111000	0.000150	0.001500	0.000015	0.015
100000	100000	10000	1200000	120000	120000	0.001500	0.001500	0.00015	0.014

$R_b/\partial b$ = 1+c'/a	$\partial R_b/\partial c'$ = 1+b/a	$\partial R_b/\partial a$ = –b×c'/a^2	$\partial R_x/\partial a$ = 1+b/c'	$\partial R_x/\partial b$ = 1+a/c'	$\partial R_x/\partial c'$ = –(a×b)/c'^{2}	u_{Rx} (ppm)	U (k = 2, ppm)
1.001	2	–0.001	1001	1001	–1000000	0.026	0.05
1.010	11	–0.1	1001	101	–100000	0.026	0.05
1.010	2	–0.01	101	101	–10000	0.026	0.05
1.100	11	–1	101	11	–1000	0.027	0.05
1.100	2	–0.1	11	11	–100	0.028	0.06

10 MΩ									
a (Ω)	b (Ω)	c' (Ω)	A(R_X) = a+b+a_x b/c' (Ω)	B = a+c'+a_x c'/b (Ω)	C(R_b) = b+c'+b_x c'/a (Ω)	ua (Ω)	ub (Ω)	uc' (Ω)	u_{Rb} (ppm)
10000	10000	10	10020000	10020	10020	0.000150	0.00015	0.00000015	0.015
10000	100000	100	10110000	10110	101100	0.000150	0.00150	0.00000150	0.015
100000	100000	1000	10200000	102000	102000	0.001500	0.00150	0.00001500	0.015
100000	1000000	10000	11100000	111000	1110000	0.001500	0.03000	0.00015000	0.030
1000000	1000000	100000	12000000	1200000	1200000	0.030000	0.03000	0.00150000	0.028

$\partial R_b/\partial b$ = 1+c'/a	$\partial R_b/\partial c'$ = 1+b/a	$\partial R_b/\partial a$ = –b×c'/a^2	$\partial R_x/\partial a$ = 1+b/c'	$\partial R_x/\partial b$ = 1+a/c'	$\partial R_x/\partial c'$ = –(a×b)/c'^{2}	u_{Rx} (ppm)	U (k = 2, ppm)
1.001	2	-0.001	1001	1001	-1000000	0.026	0.05
1.010	11	-0.1	1001	101	-100000	0.026	0.05
1.010	2	-0.01	101	101	-10000	0.026	0.05
1.100	11	-1	101	11	-1000	0.039	0.08
1.100	2	-0.1	11	11	-100	0.049	0.10

100 MΩ									
a (Ω)	b (Ω)	c' (Ω)	A(R_X) = a+b+a_x b/c' (Ω)	B = a+c'+a_x c'/b (Ω)	C(R_b) = b+c'+b_x c'/a (Ω)	ua (Ω)	ub (Ω)	uc' (Ω)	u_{Rb} (ppm)
10000	10000	1	100020000	10002	10002	0.00015	0.00015	0.000000015	0.015
10000	100000	10	100110000	10011	100110	0.00015	0.00015	0.00000015	0.015
100000	100000	100	100200000	100200	100200	0.00150	0.00150	0.0000015	0.015
100000	1000000	1000	101100000	101100	1011000	0.00150	0.03000	0.000015	0.030
1000000	1000000	10000	102000000	1020000	1020000	0.03000	0.03000	0.00015	0.030

1000000	10000000	100000	111000000	1110000	11100000	0.03000	1.00000	0.0015	0.099
10000000	10000000	1000000	120000000	12000000	12000000	1.00000	1.00000	0.0300	0.092

$\partial R_b/\partial b$ $= 1+c'/a$	$\partial R_b/\partial c'$ $= 1+b/a$	$\partial R_b/\partial a$ $= -b\times c'/a^2$	$\partial R_x/\partial a$ $= 1+b/c'$	$\partial R_x/\partial b$ $= 1+a/c'$	$\partial R_x/\partial c'$ $= -(a\times b)/c'^2$	u_{Rx} (ppm)	U (k = 2, ppm)
1.000	2	–0.0001	10001	10001	–100000000	0.026	0.05
1.001	11	–0.01	10001	1001	–10000000	0.021	0.05
1.001	2	–0.001	1001	1001	–1000000	0.026	0.05
1.010	11	–0.1	1001	101	–100000	0.037	0.07
1.010	2	–0.01	101	101	–10000	0.045	0.09
1.100	11	–1	101	11	–1000	0.115	0.23
1.100	2	–0.1	11	11	–100	0.158	0.32

1 GΩ									
a (Ω)	b (Ω)	c' (Ω)	$A(R_X) =$ $a+b+a_x$ b/c' (Ω)	B = $a+c'+a_x$ c'/b (Ω)	$C(R_b) =$ $b+c'+b_x$ c'/a (Ω)	ua (Ω)	ub (Ω)	uc' (Ω)	u_{Rb} (ppm)
10000	100000	1	1.000E9	10001.1	100011	0.00015	0.0015	0.000000015	0.015
100000	100000	10	1.000E9	100020	100020	0.0015	0.0015	0.00000015	0.015
100000	1000000	100	1.001E9	100110	1001100	0.0015	0.030	0.0000015	0.030
1000000	1000000	1000	1.002E9	1002000	1002000	0.0300	0.030	0.000015	0.030
1000000	10000000	10000	1.011E9	1011000	10110000	0.0300	1.000	0.00015	0.100
10000000	10000000	100000	1.020E9	10200000	10200000	1.000	1.000	0.0015	0.099
10000000	100000000	1000000	1.110E9	11100000	111000000	1.000	50.000	0.030	0.496
100000000	100000000	10000000	1.200E9	120000000	120000000	50.000	50.000	1.000	0.461
1000000	100000000	100000	1.101E9	1101000	110100000	0.030	50.000	0.0015	0.500

$\partial R_b/\partial b$ $= 1+c'/a$	$\partial R_b/\partial c'$ $= 1+b/a$	$\partial R_b/\partial a$ $= -b\times c'/a^2$	$\partial R_x/\partial a$ $= 1+b/c'$	$\partial R_x/\partial b$ $= 1+a/c'$	$\partial R_x/\partial c'$ $= -(a\times b)/c'^2$	u_{Rx} (ppm)	U (k = 2, ppm)
1.000	11	–0.001	100001	10001	–1000000000	0.026	0.052
1.000	2	–0.0001	10001	10001	–100000000	0.026	0.052
1.001	11	–0.01	10001	1001	–10000000	0.037	0.074
1.001	2	–0.001	1001	1001	–1000000	0.045	0.090
1.010	11	–0.1	1001	101	–100000	0.106	0.213
1.010	2	–0.01	101	101	–10000	0.144	0.287

1.100	11	–1	101	11	–1000	0.560	1.120
1.100	2	–0.1	11	11	–100	0.784	1.568
1.100	101	–10	1001	11	–10000	0.551	1.102

10 GΩ									
a (Ω)	b (Ω)	c' (Ω)	A(R_X) = a+b+a_x b/c' (Ω)	B = a+c'+a_x c'/b (Ω)	C(R_b) = b+c'+b_x c'/a (Ω)	*u*a (Ω)	*u*b (Ω)	*u*c' (Ω)	u_{Rb} (ppm)
1E5	1E5	1	1.000E10	100002	100002	0.0015	0.0015	0.000000015	0.015
1E5	1E6	10	1.000E10	100011	1000110	0.0015	0.0300	0.00000015	0.030
1E6	1E6	100	1.000E10	1000200	1000200	0.0300	0.0300	0.0000015	0.030
1E6	1E7	1000	1.001E10	1001100	10011000	0.0300	1.000	0.000015	0.100
1E7	1E7	10000	1.002E10	10020000	10020000	1.000	1.000	0.00015	0.100
1E7	1E8	100000	1.011E10	10110000	101100000	1.000	50.000	0.0015	0.500
1E8	1E8	1000000	1.020E10	102000000	102000000	50.000	50.000	0.030	0.495
1E8	1E9	10000000	1.110E10	1.11E8	1.11E9	50.000	1500.00	1.000	1.487
1E9	1E9	100000000	1.200E10	1.20E9	1.20E9	1500.00	1500.00	50.000	1.381

$\partial R_b/\partial b$ = 1+c'/a	$\partial R_b/\partial c'$ = 1+b/a	$\partial R_b/\partial a$ = –b×c'/a^2	$\partial R_x/\partial a$ = 1+b/c'	$\partial R_x/\partial b$ = 1+a/c'	$\partial R_x/\partial c'$ = –(a×b)/c'^{2}	u_{Rx} (ppm)	U (*k* = 2, ppm)
1.000	2	–0.00001	100001	100001	–10000000000	0.260	0.520
1.000	11	–0.001	100001	10001	–1000000000	0.367	0.735
1.000	2	–0.0001	10001	10001	–100000000	0.450	0.900
1.001	11	–0.01	10001	1001	–10000000	1.056	2.111
1.001	2	–0.001	1001	1001	–1000000	1.424	2.847
1.010	11	–0.1	1001	101	–100000	5.150	10.301
1.010	2	–0.01	101	101	–10000	7.148	14.296
1.100	11	–1	101	11	–1000	17.284	34.569
1.100	2	–0.1	11	11	–100	23.864	47.728

Note: c'means which add to c lead wire resistance between case GND and system GND.

5. Discussion

From the uncertainty evaluation results (Table 1), it can be seen that the calculated value given by equation (1) and the resulting uncertainty depend on the measurement accuracy and measurement uncertainty of each element resistance. First, because the element resistance c connected to the case GND in Fig. 1 (a) has a relatively small resistance value compared with elements a and b, an accurate R_x can be obtained only when the resistance c is accurately measured and substituted into equation (1). Therefore, it can be seen that when the element resistance c can be accurately measured, a measurement uncertainty of 0.1 ppm level can be obtained (Table 1). Second, if the two element resistances a and b increase, the leakage resistance effect should be considered. Thus, when the wye-delta-type high resistance was manufactured, all insulating parts of the BNC terminals and between the case and the BNC terminals were made of Teflon to minimize the leakage resistance effect. In addition, if the element resistance c is accurately measured and the error is corrected (Table 1), an uncertainty of less than 1 ppm could be obtained for all ranges from 1 MΩ to 10 GΩ. In particular, it can be shown that the uncertainty level in the 1 GΩ range shown in Table 1 was comparable with the measurement uncertainty level tried and achieved in various measurement methods at several NMIs. Therefore, we can see from the results (Table 1) that high resistances having very low uncertainties could be obtained with proper selection of the element resistances a, b, and c. Moreover, for the cases where the element resistance c was 1 kΩ, it was shown that an uncertainty of less than 0.1 ppm (k = 2) could be obtained in the range of 1 GΩ. In addition to the advantage of the small uncertainty obtained here, the wye-delta-type high resistances could be used as reference values in high-resistance measurements because the calculated value and the uncertainty of the high resistance could be directly obtained whenever necessary by measuring the element resistances. In addition, because the high-resistance value can be obtained through calculation whenever necessary, there is no need for the resistance to display long-term stability, and the uncertainty associated with it is also reduced.

6. Conclusion

High resistances in the range of 1 MΩ to 10 GΩ of the wye-delta type, which can directly obtain the resistance value through calculation, were designed, and for high resistances, the combined standard uncertainties according to the uncertainty propagation law of the ISO GUM Guide were evaluated. In particular, it was shown that an uncertainty level of 0.1 ppm can be obtained by reducing the measurement uncertainties of element resistances, measurement errors of element resistances can be connected to case GND, and leakage resistance effects due to two main element resistances can be calculated. This result suggests that the calculated value can be used as a reference value when manufacturing and measuring an actual wye-delta-type high resistance, and that the measurement uncertainty can be minimized and applied in QMT. It is thought that it can be used as reference resistors with lower measurement uncertainty than the those currently used.

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